L3. Electromagnétisme. 2ème session 2008-2009.

I. $\vec{E}_{in} \overset{(L)}{=} \vec{E}_a + \vec{E}_{m,in}$ 1 où $\vec{E}_{m,in} \overset{(M)}{=} -\dfrac{\vec{P}}{3\varepsilon_0}$ 1 ; $\vec{P} \overset{(N)}{=} \chi\,\varepsilon_0 \vec{E}_{in} = \chi\varepsilon_0\left(\vec{E}_a - \dfrac{\vec{P}}{3\varepsilon_0}\right)$ 1

$\Rightarrow \vec{P} \overset{(O)}{=} \dfrac{3\chi}{3+\chi}\,\varepsilon_0 \vec{E}_a = \dfrac{3(\varepsilon-\varepsilon_0)}{(\varepsilon+2\varepsilon_0)}\cdot\varepsilon_0\vec{E}_a$ (P) 0,5 et $\vec{p} = \vec{P}\cdot V \overset{(Q)}{=} \dfrac{4}{3}\pi R^3\,\vec{P}$.

0,5   0,5   } 4,5

II Condensateur à diélectrique (6 pts)

1. $\vec{E}\left|\begin{array}{l}\vec{E}_{in} = \dfrac{\varepsilon_0}{\varepsilon}\vec{E}_a \\ \vec{E}_{ex} = \vec{E}_a\end{array}\right.$ (1) $\quad \vec{D}\left|\begin{array}{l}\vec{D}_{in} = \varepsilon_0\vec{E}_a \\ \vec{D}_{ex} = \varepsilon_0\vec{E}_a\end{array}\right.$ (1) $\quad \vec{P}\left|\begin{array}{l}\vec{P}_{in} = \vec{D}_{in} - \varepsilon_0\vec{E}_{in} \\ \quad\;\; = \varepsilon_0\vec{E}_a\left(1-\dfrac{\varepsilon_0}{\varepsilon}\right) \\ \vec{P}_{ex} = \vec{0}\end{array}\right.$ (1)

On a bien $\vec{E}_{in} = \vec{E}_{m,in} + \vec{E}_a = -\dfrac{\vec{P}}{\varepsilon_0} + \vec{E}_a = -\underbrace{\left(\dfrac{\varepsilon}{\varepsilon_0}-1\right)}_{\chi_e = \varepsilon_r - 1}\vec{E}_{in} + \vec{E}_a$

Distribution des charges :

+ + + + + ← $\sigma_{in} = \vec{P}.\vec{n}$, $\vec{n} = \pm\vec{e}_x$ (1) $\sigma_{in} = \pm P$; Continuité de D_n.
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2. $\vec{D}_1 = \varepsilon_1\vec{E}_1$ et $\vec{D}_2 = \varepsilon_2\vec{E}_2$; $\vec{D}_1 \overset{(0,5)}{=} \vec{D}_2 = \vec{D}_{ex} \overset{(0,5)}{=} -\sigma_{ex}\,\vec{e}_x$

$U \overset{(0,5)}{=} e_1E_1 + e_2E_2 = e_1\dfrac{D_1}{\varepsilon_1} + e_2\dfrac{D_2}{\varepsilon_2} = \left(\dfrac{e_1}{\varepsilon_1}+\dfrac{e_2}{\varepsilon_2}\right)D = \left(\dfrac{e_1}{\varepsilon_1}+\dfrac{e_2}{\varepsilon_2}\right)\dfrac{Q}{ab}$

$C = \dfrac{ab}{\dfrac{e_1}{\varepsilon_1}+\dfrac{e_2}{\varepsilon_2}}$ (0,5) $= \dfrac{1}{C_1}+\dfrac{1}{C_2}$, 2 capacités en série.

III.

11 pts

1) $\vec{E}_\ell = \vec{E} + \dfrac{\vec{P}}{3\varepsilon_0}$ (A) 1, $\underbrace{\boxed{\uparrow\vec{E}\ \uparrow\vec{P}} = \boxed{\vec{P}\uparrow\ \bigcirc} + (\uparrow\uparrow)}_{(B)\ \vec{E}_\ell = E_\ell(1) + E_\ell(2)}$ 1, $\vec{E}_\ell(2) = \vec{0}$ pour amorphes (C) 1

2) $\vec{P} = \varepsilon_0\underbrace{(\varepsilon_r-1)}_{\chi}\vec{E} + N\varepsilon_0\alpha_{or}\underbrace{\left(\vec{E}+\dfrac{\vec{P}}{3\varepsilon_0}\right)}_{E_\ell}$. On définit $\alpha_{or} = \dfrac{p_0^2}{3\varepsilon_0 kT}$

(D) 1   (E) 1   (F) 1

3) $\vec{P}\left[1-\dfrac{N\alpha_{or}}{3}\right] = \varepsilon_0\vec{E}\left[\varepsilon_r - 1 + N\alpha_{or}\right] \Rightarrow \vec{P} \overset{(G)}{=} \varepsilon_0\vec{E}\,\underbrace{\dfrac{(\varepsilon_r - 1 + N\alpha_{or})}{1-\dfrac{N\alpha_{or}}{3}}}_{\chi_{total}}$ 1

$(\varepsilon_r)_{total} = 1 + \chi_{total} \overset{(H)}{=} \dfrac{1 - \dfrac{N\alpha_{or}}{3} + \varepsilon_r - 1 + N\alpha_{or}}{1-\dfrac{N\alpha_{or}}{3}}$ 1

$(\varepsilon_r)_{total} \overset{(I)}{=} \dfrac{\varepsilon_r + \frac{2}{3}N\alpha_{or}}{1-N\dfrac{\alpha_{or}}{3}} \approx \left(\varepsilon_r + \tfrac{2}{3}N\alpha_{or}\right)\left(1+\dfrac{N\alpha_{or}}{3}\right) \simeq \varepsilon_r + N\alpha_{or}\left(\dfrac{\varepsilon_r}{3}+\dfrac{2}{3}\right)$ (J) 1

1

4) Continuité de $\vec{D}$: $\varepsilon_0\vec{E}_{ex} = \varepsilon_0(\varepsilon_r)_{total}\cdot\vec{E}_{in}$

$\vec{E}_{in} = \dfrac{1}{(\varepsilon_r)_{total}}\cdot\vec{E}_{ex}$ (K) 1